

Exercise 4

Prove that if $z_1 z_2 z_3 = 0$, then at least one of the three factors is zero.

Suggestion: Write $(z_1 z_2) z_3 = 0$ and use a similar result (Sec. 3) involving two factors.

Solution

Suppose that

$$z_1 z_2 z_3 = 0.$$

Use the associative law for multiplication.

$$(z_1 z_2) z_3 = 0$$

Use the fact that if a product of two complex numbers is zero, then so is at least one of the factors.

$$z_1 z_2 = 0 \quad \text{or} \quad z_3 = 0$$

Use this fact once more.

$$z_1 = 0 \quad \text{or} \quad z_2 = 0 \quad \text{or} \quad z_3 = 0$$

Therefore, if $z_1 z_2 z_3 = 0$, then at least one of the three factors is zero.